

## **How Luca Pacioli probably satisfied himself that the Venetian method**

### **has a sound mathematical foundation**

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Pacioli and his Summa are discussed in many books and academic papers, but there does not appear to be one that explores how he would have used his algebraic skills to assure himself that the Venetian method that he so admired [1] had a sound mathematical foundation; at least, I have been unable to find such an exposition in the literature. As a mathematician of some repute I feel sure that he would have wanted to prove to himself that the Venetian method had such a foundation, before including it in his book; he was too analytical a man not to have done so.

How then did Pacioli first come into contact with the Venetian method, and why did he become proficient in using it? His life is well known so I will only briefly summarise the parts that are relevant to this paper [5].

Pacioli was only in his teens when he went to Venice to teach a prosperous merchant's three teenage sons how to be competent in commercial matters. Naturally, before he could teach bookkeeping to them, he had to learn how it worked for himself. The merchant helped him by involving him in his own business dealings. He would have learned all the rules and procedures that were in use at the time, indeed he tells us that he was very impressed with the Venetian method [6]. Not only did he teach the merchant's sons, he

also spent the next six years as the merchant's assistant, so he would have become very proficient in using the method himself. [7]

When he was 26 he became a Franciscan monk, choosing a branch that allowed him to travel, teach, and further his studies in mathematics and theology. His efforts certainly paid off because, in 1475, he became a Professor at the University of Perugia staying until 1480 and returning there in 1487 to teach for a further year.

Mathematics was a very new academic discipline and Pacioli was one of the first persons to hold a chair in it - and at a well regarded university too. It was a time of vigorous development in his subject and his own work dovetailed into it well. In 1489 he taught in Rome and in the early 1490's he taught at the University of Naples for three years.

He had been working on his famous book, the Summa, for a long time, probably from when he was teaching those teenage sons - he may even have used some of it as a text book when he was teaching them - but it was not until the printing press was developed that it became feasible for him to publish it.

Pacioli stated quite clearly that the Summa was intended to be a practical book; a compendium of what was known of mathematics at that time, with an emphasis on those parts that would help commercial people to make better decisions and record their transactions in clear and organized ways; [8]

Professor Weis [9] invented a conversation with Pacioli in order to get answers to a series of questions that he wanted to ask him. He

posed some interesting questions, some of which are relevant to this paper. He asked him:

*Q: Fra Pacioli, did you have any inkling that posterity would award you the title 'Father of Accounting'?*

*P: on the contrary. My field was mathematics .....all my manuscripts were attempts to apply mathematical principles.....I published the bookkeeping model that was used by Venetian merchants because it hadn't yet been written down in a complete coherent form.*

*Q: Just what did the bookkeeping model have to do with mathematics?*

*P: A lot. You see the Venetian method – you call it double entry – was an application of Arabic algebra..... which was a new toy.*

*Q: A new toy?*

*P: For scholars yes..... It was revolutionary.....That's how the Venetian method evolved.*

Unfortunately, Weis did not follow up this question with 'Please explain to me how you used algebra to analyse the Venetian method' and he moved on to other topics. He introduced Pacioli's friendship with Leonard da Vinci and surmised that it was only after Leonardo had read the Summa that he invited Pacioli to come to the court of Milan, to tutor him in mathematical perspective and proportion. Pacioli joined Leonardo there in 1496, beginning a seven year long relationship.

The conversational approach that Weis used appeals to me so let's imagine that we are listening to a conversation that Pacioli is having with Leonardo da Vinci about the Venetian method (the double entry method in use in Venice at that time). Leonardo has been reading his description of it recently and he has some questions he wants to pose to his friend, Luca. Let's eavesdrop:

*PACIOLI: Good morning, Leonardo, what did you think of my treatise<sup>A</sup>?*

*LEONARDO: I found it very interesting but I have some questions, as I find it a bit puzzling in places. I know that you are describing an existing method, so really my questions are aimed at why the Venetian method works in the way that it does.*

*PACIOLI: What is your first question?*

*LEONARDO: Why does the Venetian method treat assets and expenses in the same way? I would have thought that assets were good things to have whilst expenses one could do without!*

*PACIOLI: It struck me immediately that the Venetian method can be thought of as being an equation; to put it at its simplest we have:*

*Assets+ Cash = Liabilities + Capital*

*LEONARDO: That struck me too, but why not use the simpler equation 'Assets=Liabilities'?*

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<sup>A</sup> The article 'Comments on some obscure or ambiguous points in the Summa' by Hernandez-Estevé were very helpful because of the clarity of his explanations : Accounting Historians Journal, June 1994

*PACIOLI: Actually I am going to, but first let me explain why I have separated out 'capital' from the other liabilities and 'cash' from the other assets. Consider 'capital':*

*Owners tend to identify with their businesses and, since capital represents their interest in the business, they find it hard to think of it as being a liability [11]. So I need to change the way they look at things.*

*In my treatise I made the critical point that the books should be kept from the point of view of the business, not from that of the owner. So, if the owner had a business that was outside his other activities, I suggested that the business should itself be regarded as if it were a person<sup>B</sup>; the owner's servant. Because it is a servant the business cannot own any of the things that the owner supplies to it; it owes them to the owner. So, from the business's point of view, capital is a liability [12]. If the owner looks at it from this angle then I think that I can convince him that it is a liability, not of his, but of the business's as it owes it to him. Hence, in the analysis that follows I am just going to refer to liabilities, including capital as one of them.*

*Now let me explain why I have separated out cash from the other assets; let us say the owner starts a business by investing some cash in it and our first equation is:*

$$\begin{aligned} \text{Cash} &= \text{Capital} \\ 100 &= 100 \end{aligned}$$

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<sup>B</sup> In Pacioli's own (translated) words "you must imagine this store is just like a person"

*The business then converts all the cash into other assets, for example, a ship and some inventory and our equation becomes:*

$$\begin{array}{rclcl}
 \text{Ship} + \text{Inventory} + \text{Cash} & = & \text{Capital} & & \\
 & & 100 & = & 100 \\
 & +40 & -40 & & = 0 \\
 +60 & & -60 & & = 0
 \end{array}$$

*so we have:*

$$\begin{array}{rclcl}
 \text{Ship} + \text{Inventory} + \text{Cash} & = & \text{Capital} & & \\
 +60 & +40 & 0 & = & 100
 \end{array}$$

*If the business is a success then, by the end of its life, it will have converted these assets back into more cash than it started with. Let's say it created '20' units of extra cash, which, of course, it owes to its owner. Eliminating the intermediary transactions, we have:*

$$\begin{array}{rcl}
 \text{Cash} & = & \text{Capital} \\
 120 & = & 120
 \end{array}$$

*Cash, then, is different from the other assets; it is an intermediary item, being converted into other assets but not being traded itself. This is an important distinction when we consider how profit and loss is recorded in the system.*

*I will now revert to using the simpler equation 'Assets = Liabilities' but I need to expand it to include a 'profit and loss' variable. If this is a profit then it is a liability for the business as it is owed to the owner. At the start of the business there will be neither a profit nor a loss, thus I have:*

*Assets = Liabilities + profit and loss.*

*Since profit and loss is equal to revenues less expenses, I can write:*

*Assets = Liabilities + Revenues - Expenses*

*I can get rid of that minus item by adding expenses to both sides:*

*Assets + Expenses = Liabilities + Revenues - Expenses + Expenses*

*This simplifies to:*

*Assets + Expenses = Liabilities + Revenues.*

*So the Venetian method deals with assets and expenses in the same way because they are on the same side of the equation<sup>C</sup>. Actually assets, other than cash, are no different from expenses.*

*LEONARDO: What? Assets are the same as expenses? How can that be?*

*PACIOLI: Consider inventory first. The intention is to sell it for more than it cost. It cost '40' units and, say, we sell it for '50'. The equation becomes*

<i>Ship</i>	<i>+</i>	<i>Inventory</i>	<i>+</i>	<i>Cash</i>	<i>+</i>	<i>Expenses</i>	<i>=</i>	<i>Liabilities</i>	<i>+</i>	<i>Revenues</i>	
<i>60</i>	<i>+</i>	<i>40</i>		<i>0</i>	<i>+</i>	<i>0</i>	<i>=</i>	<i>100</i>	<i>+</i>	<i>0</i>	
		<i>-40</i>				<i>+40</i>					<i>= 0</i>
						<i>+50</i>	<i>=</i>			<i>+50</i>	

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<sup>C</sup> Pacioli does not explain why expenses are dealt with in the same way; he just states it in ch.22

*The '40' that was in inventory, now that it has been sold, has to be removed from inventory and it becomes an expense, as you can see. Inventory is an expense waiting to happen; only time separates them [13].*

*LEONARDO: I see, but what about the ship?*

*PACIOLI: I have not considered the ship. I am concerned only with merchants trading their goods. If there is a shipwreck then I will include the cost of the ship in extraordinary expenses and charge it to profit and loss. I am not interested in amounts that might be owing or due either. Mine is a form of cash accounting. However, if I was advising a ship owner then my argument would be as follows: The business will use the ship for many years and, if it is to calculate a profit and loss figure for every year, it will need to have a 'using up' charge, a depletion amount<sup>D</sup>, to show that a part of it has been converted into an expense. Eventually all of it will be converted that way. Again, only time separates the ship from being an expense [14].*

*LEONARDO: Thank you Luca, now, I noticed that you made the equation balance in different ways; there was a minus balanced by a plus on the same side of the equation and a plus balanced by a plus on the other side of the equation so, since in each case you made two entries, I suppose this is a double entry system, even though they balance the equation in different ways?*

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<sup>D</sup> Pacioli did not discuss depreciation but I think that he would have explained it this way, had it arisen.

*PACIOLI: Yes it is<sup>E</sup>, and there are several other different ways of preserving the balance. As you know, I introduced symbols for 'plus' and 'minus' and the concept of 'minus' has proved to be very useful [15]. For example, you have been using it when you pointed out we can have a plus and a minus on one of the sides of the equation, equaling zero, or a plus equaling a plus on both sides, or indeed a minus balancing a minus. The Venetian method only uses one of these balancing methods, a plus balanced by a minus, and it achieves this by subtracting all the liability and revenue items from both sides, thus making the equation self- balancing, like this:*

$$\text{Assets} + \text{Expenses} - \text{Liabilities} - \text{Revenues} = 0$$

*For example:*

<i>Inventory</i>	<i>+</i>	<i>Cash</i>	<i>+</i>	<i>Expenses</i>	<i>-</i>	<i>Liabilities</i>	<i>-</i>	<i>Revenues</i>	<i>= 0</i>
		<i>+40</i>		<i>-40</i>					<i>= 0</i>
		<i>-40</i>				<i>+40</i>			<i>= 0</i>
				<i>+50</i>				<i>-50</i>	<i>= 0</i>

*LEONARDO: You have introduced minus signs, yet the Venetian method does not use them, how do you explain that?*

*PACIOLI: Cash transactions are going to occur very frequently and they will involve both increases and decreases. These will occur successively and in both positive and negative amounts, all mixed up in the one column. This will make it difficult for the merchants to know how much came in and how much went out, and control of cash is a vital thing for the success of the business [16]. Trying to*

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<sup>E</sup> Ellerman states that: ‘the real choice between the double entry method and the complete single entry method of recording transactions is the choice between using unsigned (“single-sided”) numbers in “two-sided” accounts (DEB) or signed (“two-sided”) numbers in “single-sided” accounts (complete single entry accounting)

*add the positives and the negatives separately would have been prone to error, so they decided to keep 'increase' changes to the equation separate from 'decreases' by recording them as separate items for each heading, like this:*

<i>Inv<sup>+</sup></i>	<i>Inv<sup>-</sup></i>	<i>Cash<sup>+</sup></i>	<i>Cash<sup>-</sup></i>	<i>Exp<sup>+</sup></i>	<i>Exp<sup>-</sup></i>	<i>Liabs<sup>+</sup></i>	<i>liabs<sup>-</sup></i>	<i>Rev<sup>+</sup></i>	<i>Rev<sup>-</sup></i>	<i>= 0</i>
40		40								<i>= 0</i>
	40			40						<i>= 0</i>
		50						50		<i>= 0</i>

*Since each heading now has both a positive and a negative column, I do not need to put a sign between the headings themselves; they would all be plus signs and are therefore superfluous. The sign of an entry is determined by which column it is entered in, so there is no need to attach plus or minus signs to the numbers themselves.*

*So, Leonardo, this is where the rule that 'every left hand entry has to be balanced with a right hand one' comes from; it is not a convention, but a consequence of the algebra.*

*Notice that although there is no need for plus and minus signs to be attached to the entries, the clerks just have to make sure that they put the items into the right columns and under the correct headings; they do not have to understand algebra or use minus amounts – all they have to do is follow the rules, even if they do not understand why! [17]*

*There is another way of arriving at this rule too. Consider this equation:*

$$(a - b) = (a - b)$$

*If we subtract  $(a - b)$  from both sides we get*

$$(a-b) - (a-b) = 0$$

*Removing the brackets gives us*

$$a - b - a + b = 0$$

*Here a left hand 'a' is eliminated by another left hand 'a' and the opposite is true for the 'b's. This does not lead to the formulation of an easy rule for the clerks to follow. The Venetian method then takes advantage of the fact that:*

$$a - b - a + b = 0$$

*is also true if we write:*

$$a - b + b - a = 0$$

*because the order of the variables in the equation is irrelevant. Now we have a left hand entry eliminated by a right hand one, and the formulation of the rule is again, 'for every left hand entry there must be an equal right hand one'. As I said before, this rule is not a convention; it is derived from the application of algebra and arithmetic.*

*LEONARDO: I think I prefer your first explanation though. At first sight it seems more complicated but I think it explains the Venetian method in a more comprehensive way.*

*LEONARDO: I still have a few more questions, Luca*

*PACIOLI: That's fine, fire away.*

*LEONARDO: Well, why does the Venetian method call left hand entries debits and right hand ones credits, and why does it refer to debits as 'shall give' and credits as 'shall have or receive'; where did those terms come from?*

*PACIOLI: There was a need for some names, so that the clerks could be instructed; it's easier to say 'debit the account' than to say 'put it on the left of the account'. The terms come from the Latin, and their derivation takes us back to the equation and the business I described earlier. I told you that the business owes anything it holds to the owners and that the owners trust it to eventually pay them. As you know, the Latin for 'owe' is 'debeo' and for 'trust' is 'credo' [20], hence the terms 'debit' and 'credit'. The business is holding assets, so these are debit types and will be increasing on the left. The owners trust it and so these are credit types and they will increase on the right [21].*

*As regards 'shall give' and 'shall have or receive' it is easiest to understand these if you think in terms of an individual debtor or creditor. A debtor is a person who will, eventually, give to the business the amount that he owes and the creditor will eventually receive from the business the amount that he is owed [22]. These terms were introduced to help the clerks make the correct entries. However, it is less easy to apply them to expenses and revenues. Indeed there is likely to be some confusion about debits and credits when applied to expenses, revenues, and capital itself [23]. This is because the explanations I am giving to you are not recorded in my*

*treatise; they can't be, as the clerks would not understand them; instead I have to give them rules to follow, written in the everyday language that they understand [24].*

*Since left hand entries are debits and right hand one are credits the two columns under each account can be described that way so, in its algebraic form, we have:*

<i>Assets</i>	<i>Expenses</i>	<i>Liabilities</i>	<i>Revenues</i>	<i>= 0</i>
+ -	+ -	+ -	+ -	
DR CR	DR CR	DR CR	DR CR	

*I am including the plus and minus signs and the '=0' for you to see, Leonardo; they are not included in my treatise nor in the Venetian method.*

*LEONARDO: You keep referring to the equation and I see that it does enable you to explain the logic, but the Venetian method hasn't got an equation in it, so how was it got rid of?*

*PACIOLI: The equation would have become very unwieldy as more headings were included and more transactions needed to be recorded. So, in effect, it was cut up. Actually it is still there, but you cannot see it, because each heading is written on a page in a book, called a 'ledger'. Each heading is called an account. There can be more than one account on each page, of course. Each page is numbered consecutively to prevent pages being fraudulently removed or added. As each entry is made from the journal into the ledger it is referenced back to it. This allows the merchant to check to see that the entries were made accurately. It's all very clever and you can see why I am impressed by the Venetian method.*

*As I just mentioned, there is just one major rule; that 'for every debit entry there must be an equal credit entry'.*

*LEONARDO: Please explain to me now how the balance on an account is calculated, since the plus and minus entries are being made into separate columns?*

*PACIOLI: Again, the solution is very clever. In order to avoid subtraction (and minus numbers) both columns are totaled and an amount is added to the smaller of them to make them equal. [25] Naturally, this new left or right hand entry has to be balanced by putting an equal and opposite entry into a newly opened account, with the same name, in a new ledger. Not only does this identify the balance, it also sets up the accounts for the next period of trading.*

*LEONARDO: So now that we have balances identified for all asset, expense, liability and revenue accounts, how does the merchant work out if the business has made a profit or not?*

*PACIOLI: Simple again. He introduces a zero balance profit and loss account into the equation and transfers all the revenue and expense balances into it. This closes what are, in effect, temporary accounts and, if the credit column total is larger than the debit column the business will have made a profit. This can then be transferred to the capital account to close the profit and loss account as this, too, is just a temporary one [26].*

*LEONARDO: I have another question. You mentioned earlier that the clerks have to be told into which headings to enter the transactions. Is that where the journal comes in?*

*PACIOLI: Yes, it is. Each entry in the journal instructs the clerks in two ways; it tells them under which two headings to enter the items and it identifies whether the entry should be on the left or the right under each heading. The words 'per' and 'a' signify which headings to go to. The 'per' entry signifies that the entry should be on the left and the 'a' signifies that the entry should be made on the right [18]. Algebraically it does not matter in which order the 'per' or the 'a' are entered, but I have followed current practice and said that the 'per' item should be entered first. Since every pair of entries is made into positive and negative columns the equation will always balance to zero.*

*The journal is very important as it allows the owner to check that the entries into the equation have been made correctly. As you just inferred, balancing is a 'necessary' but not a 'sufficient' condition for the entries to have been made correctly. The journal takes care of the 'sufficient' requirement.*

*LEONARDO: That's terrific, Luca, but, as you just implied, the equation will still balance even if they get the entries round the wrong way or enter them in the wrong accounts, how is that dealt with in the system; does the merchant erase an incorrect one and then make the correct entry?*

*PACIOLI: No, that would open the door to fraud as well. Instead of erasing it he should put an equal amount on the opposite side and then make the correct entry in the correct account [19].*

*LEONARDO: one final question. You mention in your Summa that there are nine ways that the business can buy, or sell items, all of*

*them being a mixture of cash, barter, or time transactions [27].  
What do you mean by 'time' transactions?*

*PACIOLI: Let me explain: The business can pay cash immediately for a purchase or it can agree with the supplier to pay the cash later. Similarly it can receive cash immediately for a sale, or agree to receive it later. That is what I mean by time transactions. If the business sells on a time basis the buyer owes it cash and so is called a debtor. If the business buys on a time basis it is being trusted by the supplier who is therefore called a creditor. Earlier I explained to you that cash was different from the other assets. Now we can link its two timing differences, debtors and creditors, to this statement. From this point of view cash too is a debtor [28], not an operating asset like inventory, or a ship.*

*I also mentioned using bills of exchange [29]; this is just a method that allows the owner of the bill to get cash straight away, if he wants to. Its advantage is that it can be sold, to a bank, for example. The bank would make a charge for this, of course. The bill of exchange will be presented on its due date and the presenter will receive cash for it; it is just another time factor really.*

*LEONARDO: Thank you very much Luca, It is all much clearer to me now.*

*PACIOLI: Well, I had to convince myself that the Venetian method had a sound mathematical foundation, and it does, as I have just demonstrated to you. Unfortunately, though, as I explained earlier, there is no way that the clerks would understand algebra so I have had to record the rules in their original forms. It's a pity, but there it is; as long as they follow these rules and do not question them all*

*will be well [30]. However, I can foresee that those merchants who do not understand the rules for debits and credits will have some problems, especially when they receive statements from their banks showing them what balances they have with them.*

*LEONARDO: Why should that be a problem?*

*PACIOLI: Well, the bank will present the balance from the bank's point of view [31]. Thus they regard a merchant as an asset if he has borrowed money – he is one of their debtors and the statement will have a debit balance. If he has deposited money they regard him as a liability because they owe him money - he is one of their creditors and the statement will have a credit balance [32]. If a merchant sees a credit balance he will be happy; he has money in the bank. But if he sees a debit balance the opposite is true. Given that scenario, he will soon come to believe that credits are nice things to have whilst debits are not.*

*Finally, when he looks at his business accounts, he might have trouble understanding why assets, which he feels are nice things to have, and expenses, which are not, are dealt with in the same way in the double entry system; they both increase with debits and decrease with credits. We know why, don't we? It's because assets and expenses are on the same side of the original equation, and, in any case, assets are expenses waiting to happen. The only difference between them is a timing one.*

*Leonardo: Let me tell you a story that illustrates one of your points very nicely: I have been reading Ptolemy's Almagest, in which he describes Mars as the 'red planet', so I expect it is a very hot place and Martians (if there are any) would be used to a hot climate. Well, imagine that a Martian visited Earth and landed in the mountains in mid-winter. There was thick snow on the ground and he was*

*desperate to find shelter. So, seeing a woodsman's house he knocked on the door and was invited in. Once inside, he saw the woodsman sitting at his breakfast table, with a bowl of steaming food in front of him. He noticed that every time the man took a spoonful he would blow on it for a few seconds before eating it. The Martian asked the man's wife, "Why is he doing that?"*

*"Well, he is cooling it, because it is very hot," was the reply.*

*When he had finished his breakfast, the woodsman took his axe and went into the forest. The Martian decided to follow him, because he was interested in everything he did. He watched him swinging his axe as he cut up trees into logs for the fire. After a while, the woodsman put down his axe and began blowing on his hands. "What is he doing now?" the Martian asked the woodsman's wife, who had just joined them with some hot food. "Why, he is warming his hands because they are so cold," she replied.*

*Immediately the Martian ran away, because anyone who can blow hot and cold air out of the same mouth must be very dangerous!*

Yes, it is a pity that Pacioli did not record his algebraic approach for us. It would have saved generations of accountants having to use rules without understanding why they worked. In my case, when I queried the system many years ago, my correspondence course tutor told me "Don't ask awkward questions, do it this way because I tell you to".

Double Entry Bookkeeping is a great system, particularly now that it is available in computer packages. Of one thing I am sure; when we do get to the stars we will be taking the Venetian method with us!

## **References:**

The Pacioli quotation and page references are from the translation of the Summa by Geijsbeek 1914 reprinted by Scholars Book Co. 1974. It is available via my website from Amazon.

References 2 to 4 refer to paragraphs that have been eliminated from this simplified (i.e. non academic paper)

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3. Lerner et al: Bookkeeping and Accounting. Based on Schaum's Outline of the Theory and the Problems of Bookkeeping and Accounting: McGraw Hill 3<sup>rd</sup> Edn. P. 7 (2004)
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16. Pacioli: Ch. 23 Para. 3: `If you are in business and do not know all about it, your money will go like flies'
17. Pacioli: Ch. 1 Para. 1 and Ch. 36
18. Pacioli: Ch.12
19. Pacioli: Ch. 31
20. Geijsbeek: Discursion in Theory p14 Para. 1
21. Ibid: p15; Para. 3
22. Ibid: p 15 Para. 7
23. Ibid: pp 15-16: Stevin in conversation with the Prince
24. Pacioli: Ch. 1 Para.1 and Ch. 36
25. Pacioli: Ch. 27 and Ch. 28
26. Pacioli: Ch. 27
27. Pacioli: Ch. 9
28. Pacioli: Ch. 15 p 49 Para. 1
29. Pacioli: Ch. 19
30. Pacioli Ch. 1 and especially Ch.36
31. Pacioli: Ch 30
32. Pacioli Ch.24.p 63 Last Paragraph